Environment and Planning B: Planning and Design 2001, volume 28, pages 361 – 383

DOI:10.1068/b2697

Self-organizing pedestrian movement

Dirk Helbing
Institute for Economics and Traffic, University of Technology Dresden, 01062 Dresden, Germany
Péter Molnár
Center for Theoretical Studies of Physical Systems, Clark Atlanta University,
James P Brawley Drive, Atlanta, Georgia 30314, USA
Illés J Farkas
Department of Biological Physics, Eötvös University, Budapest, Pázmány Péter Sétány 1A,
H-1117 Hungary
Kai Bolay
Tripod Inc., 160 Water St., Williamstown, MA 01267, USA;
e-mails: helbing@rcs.urz.tu-dresden.de, pmolnar@cau.edu; fij@elte-hu, kai@bolay.de
Received 16 September 1999; in revised form 10 May 2000

Abstract. Although pedestrians have individual preferences, aims, and destinations, the dynamics of pedestrian crowds is surprisingly predictable. Pedestrians can move freely only at small pedestrian densities. Otherwise their motion is affected by repulsive interactions with other pedestrians, giving rise to self-organization phenomena. Examples of the resulting patterns of motion are separate lanes of uniform walking direction in crowds of oppositely moving pedestrians or oscillations of the passing direction at bottlenecks. If pedestrians leave footprints on deformable ground (for example, in green spaces such as public parks) this additionally causes attractive interactions which are mediated by modifications of their environment. In such cases, systems of pedestrian trails will evolve over time. The corresponding computer simulations are a valuable tool for developing optimized pedestrian facilities and way systems.

1 Introduction

Pedestrian crowds have been studied empirically for more than three decades (Carstens and Ring, 1970; Hankin and Wright, 1958; Hoel, 1968; Navin and Wheeler, 1969; Oeding, 1963; O'Flaherty and Parkinson, 1972; Older, 1968; Weidmann, 1993). The evaluation methods applied were based on direct observation, photographs, and time-lapse films. Apart from behavioral investigations (Batty, 1997; Hill, 1984) the main goal of these studies was to develop a *level-of-service concept* (Fruin, 1971; Mōri and Tsukaguchi, 1987; Polus et al, 1983), *design elements* of pedestrian facilities, (Boeminghaus, 1982; Pauls, 1984; Schubert, 1967; Whyte, 1988), or *planning guidelines* (Brilon et al, 1993; Davis and Braaksma, 1988; Kirsch, 1964; Predtetschenski and Milinski, 1971; TRB, 1985). The guidelines usually have the form of regression relations (Predtetschenski and Milinski, 1971; Sandahl and Percivall, 1972; TRB, 1985) which are, however, not very well suited for the prediction of pedestrian flows in pedestrian precincts or buildings with an exceptional architecture. Therefore a number of simulation models have been proposed, for example *queueing models* (Løvås, 1994; Roy, 1992; Yuhaski and Macgregor Smith, 1989), *transition matrix models* (Garbrecht, 1973), and *stochastic models* (Ashford et al, 1976; Mayne, 1954), which are partly related to each other. In addition, there are models for the *route choice behavior* of pedestrians (Borgers and Timmermans, 1986a; 1986b; Timmermans et al, 1992).

None of these concepts adequately takes into account the self-organization effects occurring in pedestrian crowds. These may, however, lead to unexpected obstructions due to mutual disturbances of pedestrian flows. More promising in this regard is the approach by Henderson. He conjectured that the behavior of pedestrian crowds is similar to that of gases or fluids (Henderson, 1971; 1974; Henderson and Jenkins, 1973; Henderson and Lyons, 1972). This could be partly confirmed (see section 2). However, a realistic gas-kinetic or fluid-dynamic theory for pedestrians must contain corrections due to their particular interactions (that is, avoidance and deceleration maneuvers) which of course do not conserve momentum and energy. Although such a theory can in fact be formulated (Helbing, 1992a; 1993), for practical applications a direct simulation of *individual* pedestrian motion is favorable, because a numerical solution of the fluid-dynamic equations is very difficult to obtain. As a consequence, our current research focusses on the *microsimulation* of pedestrian crowds. In this connection, a *behavioral force model* of individual pedestrian dynamics has been suggested (Helbing, 1991; 1996; 1997; Helbing and Molnár, 1995; 1997; Helbing and Vicsek, 1999; Helbing et al, 1994; Molnár, 1996) (see section 3).

A discrete forerunner of this model was proposed by Gipps and Marksjö (1985). We would also like to mention here *cellular automata* of pedestrian dynamics (Bolay, 1998; Muramatsu et al, 1999), *emergency and evacuation models* (Drager et al, 1992; Ebihara et al, 1992; Still, 1993), and *AI-based models* (Gopal and Smith, 1990; Reynolds, 1994; 1999; Schelhorn et al, 1999).

2 Observations

We have investigated pedestrian motion for several years and evaluated a number of video films. Despite the sometimes more or less 'chaotic' appearance of individual pedestrian behavior, one can find regularities, some of which become most visible in the time-lapse films such as the ones produced by Arns (1993). While describing these, we also summarize results of other pedestrian studies and observations.

(1) Pedestrians show a strong aversion to taking detours or moving opposite to the desired walking direction, even if the direct route is crowded. Consequently, pedestrians normally choose the fastest route to their next destination which has therefore the shape of a polygon (Ganem, 1998). If alternative routes are of the same length, a pedestrian prefers the one where he or she can go straight ahead for as long as possible and change direction as late as possible (provided that the alternative route is not more attractive, for example, because of less noise, more light, a friendlier environment, less waiting time at traffic lights, etc). This behavior sometimes produces 'hysteresis effects'; that is, at some locations, pedestrians tend to use a typical way to a certain point, but another way back.

(2) Pedestrians prefer to walk with an individual desired speed, which corresponds to the most comfortable (that is, least energy-consuming) walking speed (see Weidmann, 1993) so long as it is not necessary to go faster in order to reach the destination in time. The desired speeds within pedestrian crowds are Gaussian distributed with a mean value of about 1.34 m s^{-1} and a standard deviation of 0.26 m s^{-1} (Henderson, 1971).

(3) Pedestrians keep a certain distance from other pedestrians and borders (of streets, walls, and obstacles; see Brilon et al, 1993; TRB, 1985). This distance is smaller as the pedestrian hurries, and it also decreases with growing pedestrian density. Resting individuals (waiting on a railway platform for a train, sitting in a dining hall, or lying at a beach) are uniformly distributed over the available area if there are no acquaintances among the individuals. Pedestrian density increases (that is, interpersonal distances lessen) around particularly attractive places, and it decreases with growing velocity variance, for example, on a dance floor (see Helbing, 1992a). Individuals who know each other may form groups which are entities that behave in a manner similar to

single pedestrians. Group sizes are Poisson distributed (Coleman, 1964; Coleman and James, 1961).
(4) Pedestrians normally do not reflect their behavioral strategy in every situation anew but act more or less automatically (as an experienced car driver does). This becomes obvious when pedestrians cause delays or obstructions, for example, by entering an elevator or train even though others are still trying to get off.

Additionally we found that, at medium and high pedestrian densities, the motion of pedestrian crowds shows some striking similarities with the motion of gases and fluids (Arns, 1993).
(a) Footsteps of pedestrians in snow look similar to streamlines of fluids.
(b) At borderlines between opposite directions of walking one can observe 'viscous fingering' (Kadanoff, 1985).
(c) When stationary pedestrian crowds need to be crossed, the moving pedestrians form river-like streams (Arns, 1993; see figure 1).
(d) The propagation of shock waves can be found in dense pedestrian crowds which push forward (see also Virkler and Elayadath, 1994).

Apart from these phenomena, there are some similarities with granular flows.
(a) The velocity profile is flat, and the dependence of the flow on the diameter of the street does not obey the so-called Hagen – Poiseuille law (Helbing, 1992a).
(b) Similar to segregation or stratification phenomena in granular media (Makse et al, 1997; Santra et al, 1996), pedestrians spontaneously organize themselves in lanes of uniform walking direction, if the pedestrian density is high enough (Oeding, 1963; see figure 2, over).
(c) At bottlenecks (for example, corridors, staircases, or doors), the passing direction of pedestrians oscillates with a frequency that increases with the width and shortness of the bottleneck. This is analogous to the granular 'ticking hour glass' (Pennec et al, 1996; Wu et al, 1993).

In the following sections we will focus on the explanation of self-organization phenomena such as the ones mentioned above.

Figure 1. The long-exposure photograph of a stationary crowd in front of a cinema shows that crossing pedestrians form a river-like stream (reproduced with kind permission of Thomas Arns).

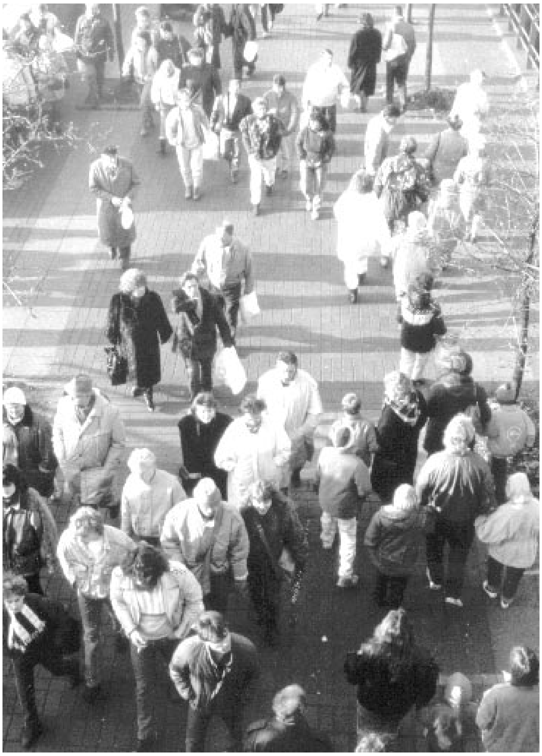


Figure 2. At sufficiently high densities, pedestrians form lanes of uniform walking direction.

3 The behavioral force model of pedestrian motion

Many people have the feeling that human behavior is 'chaotic' or at least very irregular. This may be true for behaviors in complex situations. In standard situations, however, individuals will usually not take complicated decisions between various possible alternative behaviors, but apply an optimized behavioral strategy, which has been learned over time by trial and error. Hence a pedestrian will react to obstacles, other pedestrians, etc, in a somewhat automatic way. This is comparable with the behavior of an experienced driver who usually reacts automatically to a given traffic situation without thinking about the detailed actions to be taken.

The optimal pedestrian behavior can in principle be determined by simulating the learning behavior of pedestrians. This has recently been done with a model containing several parameters which reflect the specific behavioral strategy of a pedestrian. The parameters were (randomly) changed in the simulation, and the inverse travel times as well as the collision rates with the different behavioral strategies were compared with each other. Successful strategies were replicated and further refined over time, whereas slow strategies or strategies involving frequent collisions were eliminated from the simulation. The chosen procedure was similar to evolutionary algorithms (Klockgether and Schwefel, 1970; Rechenberg, 1973) (see figure 9, below). After some time it yields a parameter set which does not change any more and reflects the optimal pedestrian behavior as regards interaction strength, acceleration behavior, etc. For example, our simulation results indicated that pedestrians develop an asymmetric avoidance behavior with respect to the right-hand side and the left-hand side, because they profit from it (Bolay, 1998; see figure 3). In fact, empirical observations of pedestrian streams show that pedestrians have a preferred side (Older, 1968; Weidmann, 1993). In Germany,

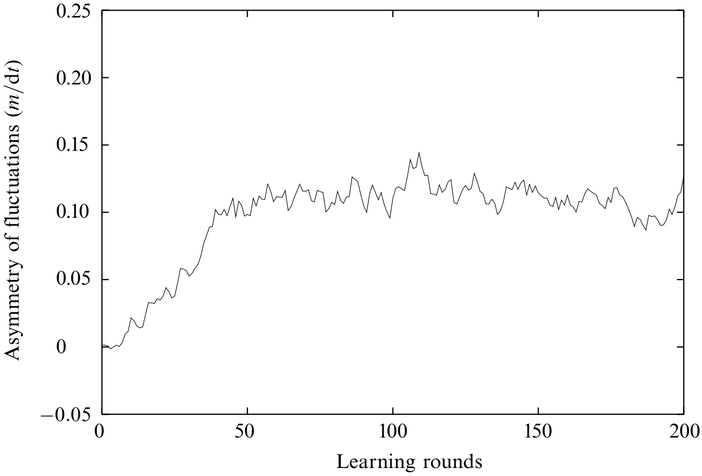


Figure 3. Assuming that pedestrians learn improved strategies by trial and error, our simulations show the evolution of an asymmetry in the avoidance behavior with respect to the left-hand and right-hand sides.

for example, it is the right-hand side. A theoretical explanation for this based on evolutionary game theory was suggested by Helbing (1991; 1993; 1995; 1997).

Assuming that pedestrians behave according to the optimal strategy (which is normally a good approximation but less true for children and tourists) we can still not predict the spatiotemporal movement of a single pedestrian. This is not only due to behavioral variations ('fluctuations' in behavior), but also because we usually do not know the destinations and preselected route of a pedestrian. Nevertheless it is possible to predict pedestrian streams with a surprisingly high accuracy. For reliable simulations of pedestrian crowds we do not need to know whether a certain pedestrian, say, turns to the right at the next intersection. It is sufficient to have a good estimate of the percentage of pedestrians turning to the right. This can be either measured empirically or calculated by means of a route choice model like the one proposed by Borgers and Timmermans (1986a; 1986b). In some sense, the uncertainty about the individual behaviors is averaged out at the macroscopic level of description, as in fluid dynamics. It is therefore not surprising that fluid-dynamic theories have been suggested for pedestrian streams (Helbing, 1992a; 1993; Henderson, 1974).

In the following we will apply an approach for modeling behavioral changes suggested by Lewin (1951). According to him, behavioral changes are guided by so-called *social fields* or *social forces*, an idea which has been put into mathematical terms by Helbing (1994; 1995). A good example is vehicle dynamics (Helbing and Tilch, 1998) or pedestrian motion (Helbing, 1991; 1996; 1997; Helbing and Molnár, 1995; 1997; Helbing and Vicsek, 1999; Helbing et al, 1994; Molnár, 1996). As the positions of the pedestrians α can be represented by points $\boldsymbol{r}_\alpha(t)$ in space, which change continuously over time t, pedestrians dynamics can be described by the following *equation of motion*:

$$\frac{\mathrm{d}\boldsymbol{r}_\alpha(t)}{\mathrm{d}t} = \boldsymbol{v}_\alpha(t) \ . \qquad (1)$$

The functions delineating the temporal changes of the actual pedestrian velocities $\boldsymbol{v}_\alpha(t)$ can be interpreted as the driving forces of this motion, which are called *behavioral forces* or *social forces* (for details see Helbing, 1991; 1993; 1994; 1995; 1997). If the behavioral force $\boldsymbol{f}_\alpha(t)$ represents the different systematic influences (of the environment and other pedestrians) on the behavior of a pedestrian α, and the fluctuation term $\xi_\alpha(t)$

reflects random behavioral variations (arising from accidental or deliberate deviations from the optimal strategy of motion), we have the following *equation of acceleration*:

$$\frac{d\boldsymbol{v}_\alpha}{dt} = \boldsymbol{f}_\alpha(t) + \xi_\alpha(t) \ . \tag{2}$$

As in physics, the behavioral force $\boldsymbol{f}_\alpha(t)$ is assumed to be the sum of several force terms which correspond to the different influences simultaneously affecting the behavior of pedestrian α. We will take into account an acceleration force $\boldsymbol{f}_\alpha^0(\boldsymbol{v}_\alpha)$, repulsive effects $\boldsymbol{f}_{\alpha B}(\boldsymbol{r}_\alpha)$ due to boundaries, repulsive interactions $\boldsymbol{f}_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha, \boldsymbol{r}_\beta, \boldsymbol{v}_\beta)$ with other pedestrians β, and attraction effects $\boldsymbol{f}_{\alpha i}(\boldsymbol{r}_\alpha, \boldsymbol{r}_i, t)$:

$$\boldsymbol{f}_\alpha(t) = \boldsymbol{f}_\alpha^0(\boldsymbol{v}_\alpha) + \boldsymbol{f}_{\alpha B}(\boldsymbol{r}_\alpha) + \sum_{\beta(\neq\alpha)} \boldsymbol{f}_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha, \boldsymbol{r}_\beta, \boldsymbol{v}_\beta) + \sum_i \boldsymbol{f}_{\alpha i}(\boldsymbol{r}_\alpha, \boldsymbol{r}_i, t) + \xi_\alpha(t) \ . \tag{3}$$

The individual force terms are discussed below.

(1) Each pedestrian wants to walk with an individual *desired speed* v_α^0 in the direction $\boldsymbol{e}_\alpha$ of his or her next destination. Deviations of the *actual velocity* $\boldsymbol{v}_\alpha$ from the *desired velocity* $\boldsymbol{v}_\alpha^0 = v_\alpha^0 \boldsymbol{e}_\alpha$ because of disturbances (by obstacles or avoidance maneuvers) are corrected within the so-called *relaxation time* τ_α:

$$\boldsymbol{f}_\alpha^0(\boldsymbol{v}_\alpha) = \frac{1}{\tau_\alpha}(v_\alpha^0 \boldsymbol{e}_\alpha - \boldsymbol{v}_\alpha) \ . \tag{4}$$

In order to compensate for delays, the desired speed $v_\alpha^0(t)$ is often increased over time, because it is given by the remaining distance divided by the remaining time until the desired destination is reached. (For example, if someone recognizes that he or she has forgotten the theater tickets, so that the remaining distance suddenly increases by the additional way home and back to the present place, the desired velocity and actual walking speed will increase discontinuously!)

(2) Pedestrians keep some distance from borders in order to avoid the risk of getting hurt. The closer the border, the more uncomfortable a pedestrian feels. This effect can be described by a repulsive force $\boldsymbol{f}_{\alpha B}$ which decreases monotonically with the distance $\|\boldsymbol{r}_\alpha - \boldsymbol{r}_B^\alpha\|$ between the place $\boldsymbol{r}_\alpha(t)$ of pedestrian α and the nearest point $\boldsymbol{r}_B^\alpha$ of the border. In the simplest case, this force can be expressed in terms of a repulsive potential V_B:

$$\boldsymbol{f}_{\alpha B}(\boldsymbol{r}_\alpha) = -\nabla_{\boldsymbol{r}_\alpha} V_B(\|\boldsymbol{r}_\alpha - \boldsymbol{r}_B^\alpha\|) \ . \tag{5}$$

Similar repulsive force terms $\boldsymbol{f}_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\alpha, \boldsymbol{r}_\beta, \boldsymbol{v}_\beta)$ are used to indicate that each pedestrian α keeps a situation-dependent distance from the other pedestrians β. This reflects the tendency to respect a *private sphere* (*territorial effect*) and helps to avoid collisions in cases of sudden velocity changes. Note that these repulsive forces are not symmetric, because pedestrians rarely react to the situation behind them. Moreover, they are strongly 'anisotropic', as pedestrians need more space in the walking direction than in a direction perpendicular to it. Usually the space requirements for the next step are taken into account. As a consequence, the repulsive interaction forces are also velocity-dependent. However, all of this is not essential for the resulting self-organization phenomena.

(3) Pedestrians show a certain joining behavior. For example, families, friends, or tourists often move in groups. In addition, pedestrians are sometimes attracted by window displays, sights, special performances (street artists), or unusual events at places $\boldsymbol{r}_i$. Both situations can be modelled by (often temporally decaying) attractive forces $\boldsymbol{f}_{\alpha i}(\boldsymbol{r}_\alpha, \boldsymbol{r}_i, t)$ in a similar way as the repulsive effects, but with an opposite sign and a longer range of interactions.

For a more detailed discussion and a concrete mathematical specification of the force terms, see Helbing (1996; 1997), Helbing and Molnár (1995), Helbing et al (2000), and Molnár (1996). However, we found that the collective phenomena occurring in pedestrian crowds (see section 3.2) are not very sensitive to the concrete specification of the forces or the model parameters (Helbing and Vicsek, 1999).

3.1 Equilibria between different forces

Some of the observations discussed in point (3) of section 2 can be easily explained by looking at equilibria between certain behavioral forces, for which the acceleration $d\boldsymbol{v}_\alpha/dt$ vanishes. For example, if we focus on pedestrians waiting on a railway platform, sitting in a restaurant, or lying on a beach, we find that the individual velocities $\boldsymbol{v}_\alpha$ are all zero. Therefore, in the absence of special attractions, the positions $\boldsymbol{r}_\alpha$ of the pedestrians follow form

$$\sum_{\beta(\neq\alpha)} \boldsymbol{f}_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{0}, \boldsymbol{r}_\beta, \boldsymbol{0}) = \boldsymbol{0} \ . \tag{6}$$

As the repulsive forces of different pedestrians are usually comparable, we get more or less equal distances between them, in agreement with the empirically observed uniform distribution. Therefore the distances between stationary pedestrians are determined mainly by the available area which is to be shared by them.

In case of additional attraction effects $\boldsymbol{f}_{\alpha i}$ (such as the stage in a rock concert), equation (6) must be supplemented by the corresponding forces of attraction:

$$\sum_{\beta(\neq\alpha)} \boldsymbol{f}_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{0}, \boldsymbol{r}_\beta, \boldsymbol{0}) + \sum_i \boldsymbol{f}_{\alpha i}(\boldsymbol{r}_\alpha, \boldsymbol{r}_i) = \boldsymbol{0} \ . \tag{7}$$

In accordance with observations, this clearly leads to more crowded regions close to the attractions at places $\boldsymbol{r}_i$. In particular, one can reproduce the observed phenomenon of people being attracted to the center of a stage.

Focusing on situations where a pedestrian α cannot overtake a slower pedestrian β moving with velocity $\boldsymbol{v}_\beta$, we have to set up the equation for the equilibrium between the acceleration force $\boldsymbol{f}_\alpha^0$ and the repulsive force $\boldsymbol{f}_{\alpha\beta}$. Because $\boldsymbol{v}_\alpha = \boldsymbol{v}_\beta$, this yields

$$\frac{1}{\tau_\alpha}(v_\alpha^0 \boldsymbol{e}_\alpha - \boldsymbol{v}_\beta) + \boldsymbol{f}_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{v}_\beta, \boldsymbol{r}_\beta, \boldsymbol{v}_\beta) = \boldsymbol{0} \ , \tag{8}$$

where the actual velocity $\boldsymbol{v}_\beta$, the distance vector $(\boldsymbol{r}_\beta - \boldsymbol{r}_\alpha)$, and the repulsive force $\boldsymbol{f}_{\alpha\beta}$ point in the desired direction $\boldsymbol{e}_\alpha$ of walking. We therefore have

$$\frac{v_\alpha^0 - v_\beta}{\tau_\alpha} + \boldsymbol{f}_{\alpha\beta}(\boldsymbol{r}_\alpha, v_\beta \boldsymbol{e}_\alpha, \boldsymbol{r}_\alpha + \Delta r_{\alpha\beta} \boldsymbol{e}_\alpha, v_\beta \boldsymbol{e}_\alpha) = 0 \ , \tag{9}$$

where τ_α denotes the 'acceleration time' to reach the desired velocity, so that $(v_\alpha^0 - v_\beta)/\tau_\alpha$ reflects the acceleration strength. From this formula (and remembering that the repulsive force increases with decreasing distance) it can be seen that pedestrians keep a smaller distance $\Delta r_{\alpha\beta} = \|\boldsymbol{r}_\beta - \boldsymbol{r}_\alpha\|$, the larger the difference between their own desired velocity v_α^0 and the speed v_β of the preceding pedestrian. This corresponds to the well-known pushing behavior of pedestrians.

Combined with the growth of the desired speed owing to delays, an interesting phenomenon is observed in pedestrian queues (Helbing, 1991; 1992b), as illustrated by figure 4 (see over).

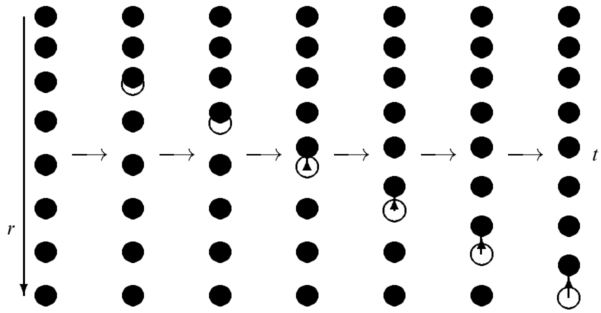


Figure 4. When the front of a queue (top) is stopped, one can often observe the following phenomenon. After some time, one of the waiting pedestrians begins to move forward a little bit, because his or her desired velocity grows, leading to a smaller equilibrium distance to the pedestrian in front. This causes the successor to follow up and so forth, leading to a wave-like propagation of the gap to the end of the queue and to a compaction of the queue (which decreases the comfort of waiting). Thereby, the tendencies of all individuals to move forward a little (in accordance with their steadily decreasing preferred equilibrium distance) add up towards the end of the queue, giving rise to larger following-up distances. A more detailed explanation of this phenomenon considering the minimal length of pedestrian strides is presented in Helbing (1991).

Finally, by looking at the equilibrium between the acceleration force $\boldsymbol{f}_\alpha^{\,0}$ and an attracting force $\boldsymbol{f}_{\alpha i}$, we can calculate how long a pedestrian joins an attraction at place $\boldsymbol{r}_i$:

$$\frac{v_\alpha^0(t)\boldsymbol{e}_\alpha - \boldsymbol{0}}{\tau_\alpha} + \boldsymbol{f}_{\alpha i}(\boldsymbol{r}_\alpha, \boldsymbol{r}_i, t) \;=\; \boldsymbol{0} \;. \tag{10}$$

In the course of time, the desired speed $v_\alpha^0(t)$ will increase, whereas the interest in the attraction (that is, the magnitude of $\|\boldsymbol{f}_{\alpha i}\|$) tends to decrease. Thus, at a specific moment $t = t_{\alpha i}$, the attractiveness $\boldsymbol{f}_{\alpha i}$ of place $\boldsymbol{r}_i$ will be compensated for by the tendency $\boldsymbol{f}_\alpha^{\,0}$ to move ahead.

Equilibrium considerations are also useful for specifying the model parameters appropriately, because certain plausibility criteria must be met. For example, pedestrians should normally not move opposite to their desired walking directions. This implies that

$$\frac{v_\alpha^0}{\tau_\alpha} \approx \frac{v_\beta^0}{\tau_\beta} \;. \tag{11}$$

3.2 Simulation of self-organizing pedestrian crowds

The behavioral force model of pedestrian dynamics has been simulated on a computer for a large number of interacting pedestrians confronted with different situations. Despite the fact that the proposed model is very simple, it describes a number of observed phenomena very realistically. Under certain conditions the *self-organization* of collective behavioral patterns can be found, just as in some related models of bird swarms (Reynolds, 1987; Vicsek et al, 1995). 'Self-organization' means that these patterns are not externally planned, prescribed, or organized, for example, by traffic signs, laws, or behavioral conventions. Instead the spatiotemporal patterns emerge through the nonlinear interactions of pedestrians. Our model (according to which individuals behave somewhat automatically) can explain the self-organized patterns described below without assuming strategic considerations, communication, or the imitative behavior of pedestrians. All these collective patterns of motion are symmetry-breaking

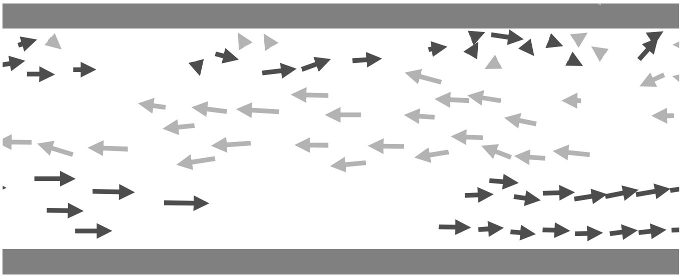


Figure 5. In crowds of oppositely moving pedestrians, one can observe the formation of varying lanes consisting of pedestrians with the same desired direction of motion (compare figure 2). This is also the case if interacting pedestrians avoid each other with the same probability on the right-hand side and on the left-hand side. The reason for lane formation is the related decrease in the frequency of necessary deceleration and avoidance maneuvers, which increases the efficiency of the pedestrian flow. (The positions, directions, and lengths of the arrows represent the places, walking directions, and speeds of pedestrians.)

phenomena, although the model was formulated completely symmetrically with respect to the right-hand and left-hand sides.

(1) Our simulations reproduce the empirically observed formation of lanes consisting of pedestrians with the same desired walking direction (Helbing, 1996; 1997; Helbing and Molnár, 1995; 1997; Helbing and Vicsek, 1999; Helbing et al, 1994; Molnár, 1996) (see figure 5). These lanes are dynamically varying. Their number depends on the width of the street and on pedestrian density.

In the conventional interpretation of lane formation, it is assumed that pedestrians tend to walk on the side which is prescribed in vehicular traffic. However, our model can explain lane formation even without assuming a preference for *any* side. The mechanism of lane formation can be understood as follows (Helbing et al, 2000): pedestrians moving against the stream or in areas of mixed directions of motion will have frequent and strong interactions. In each interaction, the encountering pedestrians move a little aside in order to pass each other. This sidewards movement tends to separate oppositely moving pedestrians. Moreover, pedestrians moving in uniform lanes will have very rare and weak interactions. Hence the tendency to break up existing lanes is negligible, when the fluctuations are small. Furthermore, the most stable configuration corresponds to a state with a minimal interaction rate and is related to a maximum efficiency of motion (Helbing and Vicsek, 1999).

Note that, at sufficiently high pedestrian densities, lanes are destroyed by increasing the fluctuation strength (which is analogous to the temperature). This gives rise to the formation of blocked situations, which may even have a regular (that is, 'crystallized' or 'frozen') structure. We call this surprising transition 'freezing by heating' (Helbing et al, 2000) (see figure 6, over) and believe that it is relevant to situations involving pedestrians under extreme conditions (panics): Imagine a very smoky situation, caused by fire, in which people do not know which is the right way to escape. When panicking, people will just try to get ahead, with a reduced tendency to follow a certain direction. Thus fluctuations will be very large, which can lead to fatal blockages. As a consequence, models for everyday pedestrian streams are not suitable for the realistic simulation of emergency situations. The latter requires simulations with modified parameter sets corresponding to less optimal pedestrian behavior. In particular, the fluctuation strength is considerably higher for panic situations than for usual conditions.

(2) In our simulations of narrow passages, we observed oscillatory changes in the walking direction (Helbing, 1996; 1997; Helbing and Molnár, 1995; 1997; Helbing et al, 1994; Molnár, 1996) (see figure 7). The conventional interpretation for a change in the walking direction is that, after some time, a pedestrian gives precedence to a

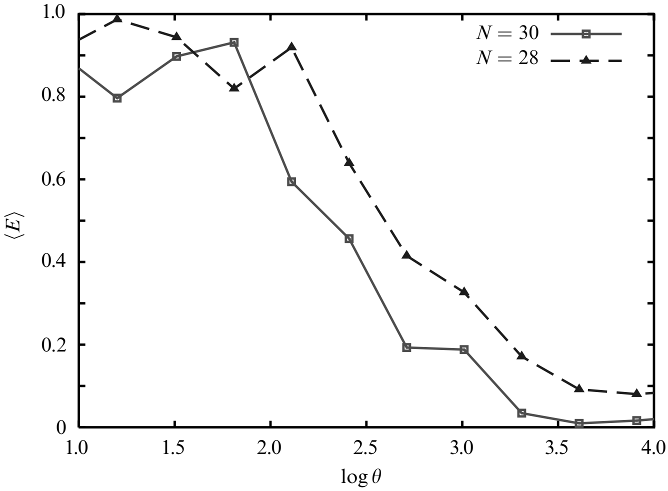


Figure 6. The ensemble-averaged efficiency $\langle E \rangle$ of the system [see equation (12)] as a function of the particle number N and the noise intensity θ on a logarithmic scale. Shown above are averages over twenty-five simulation runs with different random seeds. The decrease in efficiency from values close to 1 to values around 0 with increasing fluctuation intensity θ but a constant number N of particles indicates the transition from the fluid to the crystallized state that we call 'freezing by heating'.

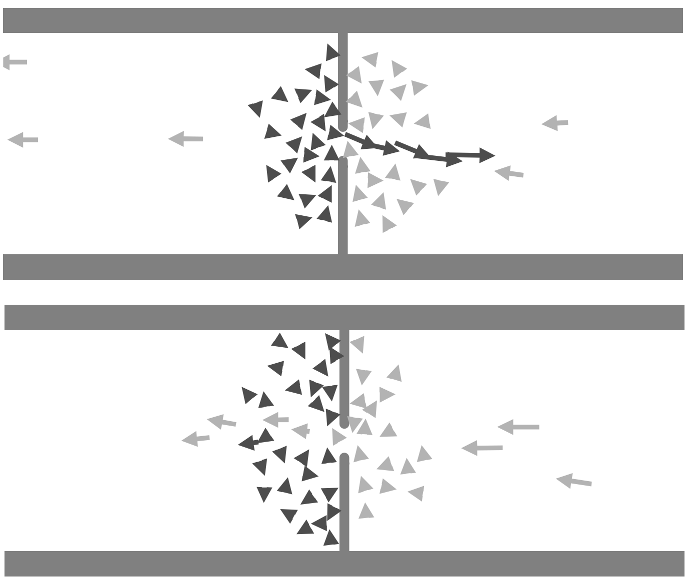


Figure 7. At narrow passages one finds an oscillation of the passing direction. When a pedestrian is able to pass through the door, normally another pedestrian can follow him or her easily (above). However, the pedestrian stream arising in this way will stop after some time owing to the pressure from the other side of the passage. Some time later, a pedestrian will pass through the door in the opposite direction, and the process continues as outlined before (below).

waiting pedestrian walking in the opposite direction. This cannot, however, explain the increase in oscillation frequency with passage width.

The mechanism leading to alternating flows is as follows: once a pedestrian is able to pass the narrowing (door, staircase, etc), pedestrians with the same walking direction can easily follow, which is particularly clear for long passages. In this way, the number and 'pressure' of waiting and pushing pedestrians becomes less than on the other side of the narrowing where, consequently, the chance to occupy the passage grows. This leads to a *deadlock* situation after some time which is followed by a change in the walking direction. Capturing the passage is easier if it is broad and short so that the walking direction changes more frequently.

(3) At intersections our simulations show the temporary emergence of unstable roundabout traffic (Helbing, 1996; 1997; Helbing and Molnár, 1997; Helbing et al, 1994; Molnár, 1996) (see figure 8). This is similar to the emergent rotation in the case of self-driven particles (Duparcmeur et al, 1995). However, the rotation direction of circular pedestrian flows is alternating. Roundabout traffic is connected with small detours but decreases the frequency of necessary deceleration, stopping, and avoidance maneuvers considerably, so that pedestrian motion becomes more efficient on average.

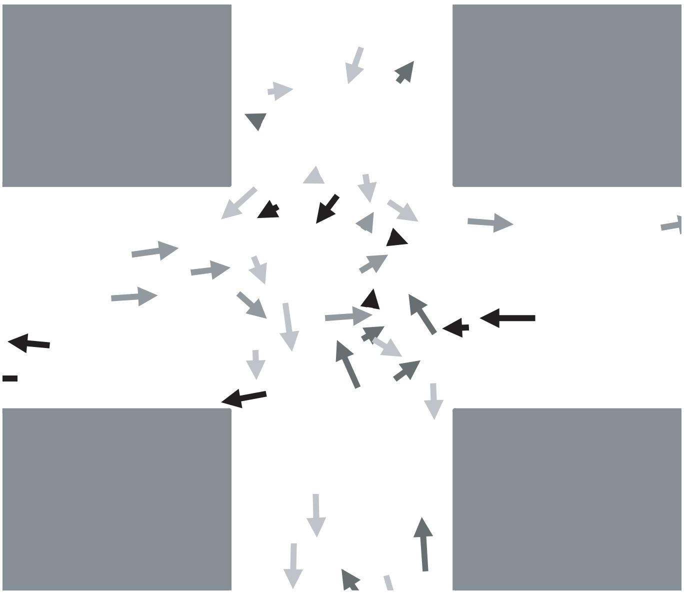


Figure 8. At intersections one is confronted with various alternating collective patterns of motion which are very short-lived and unstable. For example, phases of temporary roundabout traffic (above) alternate with phases during which the intersection is crossed in the 'vertical' or the 'horizontal' direction. The efficiency of pedestrian flow can be increased considerably by putting an obstacle in the center of the intersection, because this favors the smooth roundabout traffic compared with the competing, inefficient patterns of motion.

3.3 Optimization of pedestrian facilities

The emerging pedestrian flows depend decisively on the geometry of the boundaries. They can be simulated on a computer as early as in the planning phase of pedestrian facilities. Their configuration and shape can be varied systematically, for example, by means of evolutionary algorithms (Baeck, 1996; Bolay, 1998; Rechenberg, 1973)

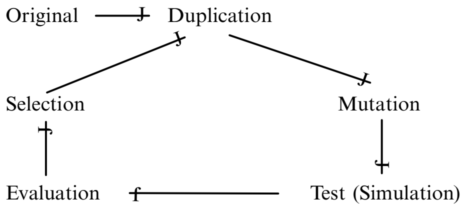


Figure 9. Illustration of evolutionary algorithms. An original configuration is duplicated. The copies are then mutated, and their performance is evaluated via simulations. The best copies are selected, mutated, etc, until the configuration has reached an optimal or at least sufficiently good performance. An example for the evolutionary optimization of a bottleneck is presented in figure 12.

(see figure 9), and evaluated on the basis of particular mathematical performance measures (Helbing, 1997; Helbing and Molnár, 1997). For example, the efficiency measure

$$E = \frac{1}{N}\sum_{\alpha} \frac{\overline{\boldsymbol{v}_\alpha \boldsymbol{e}_\alpha}}{v_\alpha^0} , \qquad 0 \leqslant E \leqslant 1 , \tag{12}$$

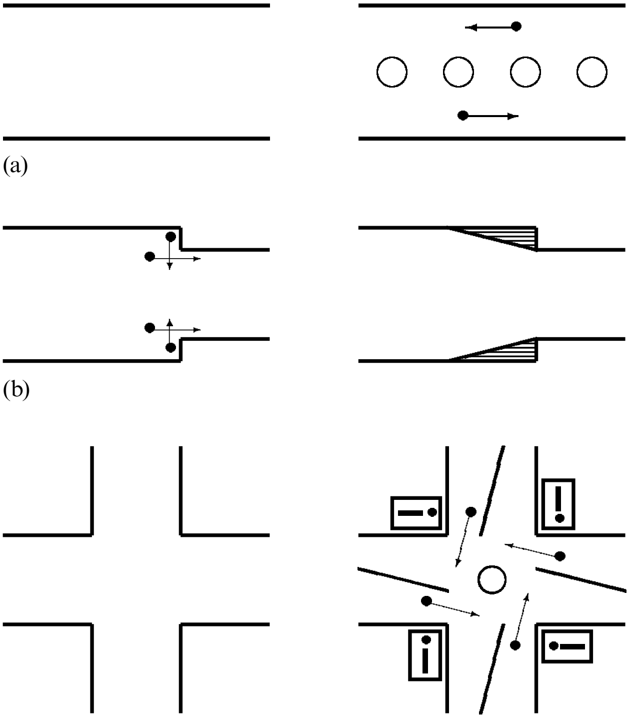
(where N is the number of pedestrians α and the bar denotes a time average) describes the average fraction of the desired speed with which pedestrians actually approach their destinations. The measure of discomfort D,

$$D = \frac{1}{N}\sum_{\alpha} \frac{\overline{(\boldsymbol{v}_\alpha - \overline{\boldsymbol{v}_\alpha})^2}}{\overline{(\boldsymbol{v}_\alpha)^2}} = \frac{1}{N}\sum_{\alpha}\left(1 - \frac{\overline{\boldsymbol{v}_\alpha}^{\,2}}{\overline{(\boldsymbol{v}_\alpha)^2}}\right), \qquad 0 \leqslant D \leqslant 1 , \tag{13}$$

reflects the frequency and degree of sudden velocity changes, that is, the level of discontinuity of walking because of necessary avoidance maneuvers. Hence the optimal configuration as regards pedestrian requirements is the one with the highest values of efficiency and comfort, $C = 1 - D$.

During the optimization procedure, some or all of the following can be varied:
(1) the location and form of planned buildings;
(2) the arrangement of walkways, entrances, exits, staircases, elevators, escalators, and corridors;
(3) the shape of rooms, corridors, entrances, and exits;
(4) the function and time schedule of room usage.

The proposed optimization procedure can be applied not only to the design of new pedestrian facilities but also to a reduction of existing bottlenecks by suitable modifications. Here we discuss four simple examples of how to improve some standard elements of pedestrian facilities (Helbing, 1997) (see figure 10).
(1) At high pedestrian densities, the lanes of uniform walking direction tend to disturb each other: impatient pedestrians try to use any gap for overtaking, which often leads to subsequent obstructions of the opposite walking direction. The lanes can be stabilized by series of trees or columns in the middle of the road [see figure 10(a)] which, when looking in the walking direction, resembles a wall (see figure 11). It also requires some detour to reach the other side of the permeable wall, which makes it less attractive to use gaps in the opposite pedestrian stream.
(2) The flow at bottlenecks can be improved by a funnel-shaped construction [see figure 10(b)] which, at the same time, saves expensive space. Interestingly the optimal form resulting from an evolutionary optimization is convex (Bolay, 1998) (see figure 12, over).



(a)

(b)

(c)

Figure 10. Conventional (left) and improved (right) elements of pedestrians facilities: (a) ways, (b) bottlenecks, and (c) intersections. The exclamation marks stand for attraction effects (for example, interesting posters above street level). Empty circles represent column or trees, and full circles with arrows symbolize pedestrians and their walking directions.

Figure 11. This photograph of a pedestrian tunnel connecting two subways in Budapest at Deák tér illustrates that a series of columns acts like a wall and stabilizes lanes (by preventing the lane width from exceeding half of the total width of the walkway).

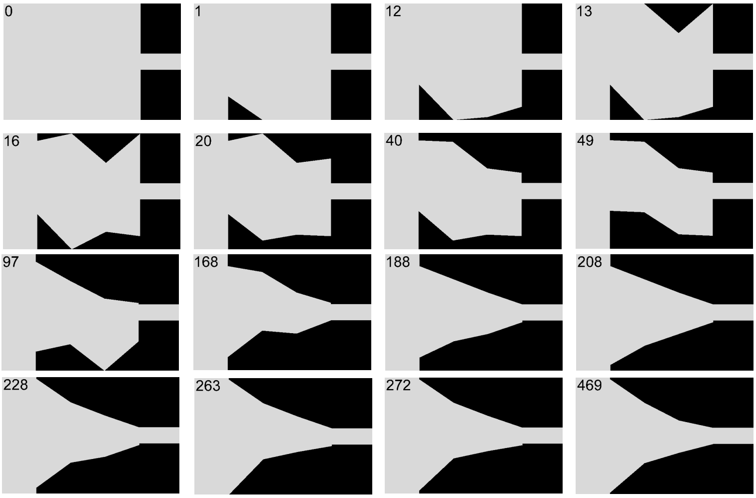


Figure 12. Different phases in the evolutionary optimization of a bottleneck.

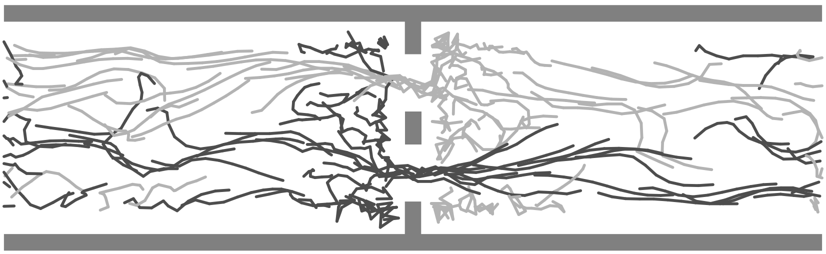


Figure 13. If two alternative passageways are available, pedestrians walking in opposite directions use different doors as a result of self-organization.

(3) A broader door does not necessarily lead to a proportional increase in pedestrian flow through it. Instead it may lead to more frequent changes of the walking direction which are connected with temporary deadlock situations. Therefore two doors are much more efficient than one single door of double width. By self-organization, each door is used for one walking direction (Helbing, 1997; Helbing and Molnár, 1997; Molnár, 1996) (see figure 13).

(4) Oscillatory changes in the walking direction and periods of standstill in between also occur when different flows *cross* each other. The loss of efficiency caused by this can be reduced by psychological guiding measures or railings initializing roundabout traffic [see figure 10(c)]. Roundabout traffic can be reduced and stabilized by planting a tree in the middle of a crossing, which suppresses the phases of 'vertical' or 'horizontal' motion in the intersection area. In our simulations this increased efficiency by up to 13%.

The complex interaction between various flows can lead to completely unexpected results owing to the nonlinearity of dynamics. [A very impressive and surprising result of evolutionary form optimization is presented by Klockgether and Schwefel (1970).] This means that planning of pedestrian facilities with conventional methods does not always guarantee the avoidance of big jams, serious obstructions, and catastrophic blockages (especially in emergency situations). In contrast, a skilful flow optimization not only enhances efficiency but also saves space that can be used for kiosks, benches, or for other purposes (Helbing, 1997).

4 Trail formation

Another interesting collective effect of pedestrian motion, which has been investigated recently, is the development of trail systems on deformable ground. A theory of human trail formation in green areas such as public parks must be able to answer the following questions. Why do pedestrians sometimes build trails in order to save 3 to 5 m, but in other cases accept detours which are much larger? How and by which mechanism do trail systems evolve in space and time? Why do trails reappear at the same places, even if they were destroyed? How should urban planners design public way systems so that walkers actually use them? Empirical investigations of these questions have, for example, been carried out by Humpert et al (1996) and Schaur (1991).

(a)

(b)

Figure 14. These examples of (a) human trails and (b) animal trails show that trails are compromises. They often deviate from the direct way where they meet other trails. Relative detours of up to 25% seem to be acceptable.

Figure 15. The splitting of a trail which meets another way (and thereby gives rise to an island in the middle) is a rather typical phenomenon.

It is known that many way systems and even streets originated from human or animal trails. For reasons of easier orientation, these mostly point to optically significant places (Humpert et al, 1996). However, the pedestrians' tendency to take the fastest route to these places (Ganem, 1998) and the specific properties of the terrain are often insufficient to explain the trail characteristics. Although trails trivially serve as shortcuts, they frequently do not offer the shortest way to the given destination, which is surprising. Detours of up to about 25% seem to be acceptable to pedestrians. Detailed studies indicate that trails are often compromises between ways which point in different directions (Helbing, 1997; 1998; Helbing et al, 1997a; 1997b) (see figure 14). For example, one often observes a splitting of trails just before they meet another way in a more or less perpendicular direction (Helbing, 1998) (see figure 15). It would be of greater consequence if trails formed a triangular, direct way system between three entry points and destinations. However, the minimal angle between splitting ways is found to be about 30°.

4.1 The active walker model

To simulate the typical features of trail systems, the aforementioned behavioral force model has been extended to an active walker model (Helbing, 1997; 1998; Helbing et al, 1994; 1997a; 1997b; Keltsch, 1996; Molnár, 1996). Like random walkers, active walkers are subject to the fluctuations and influences of their environment. However, they are also able to change their environment, for example, by altering an environmental potential, which in turn influences their further movement and their behavior. In particular, changes produced by some walkers can influence other walkers. Hence this nonlinear feedback can be interpreted as an indirect interaction between the active walkers via environmental changes, which may lead to the self-organization of large-scale spatial structures.

First, let us specify how pedestrian motion changes environment. For this, we represent the ground structure at place $\boldsymbol{r}$ and time t by a function $\mathrm{G}(\boldsymbol{r}, t)$ which reflects the comfort of walking. Trails are characterized by particularly large values of G. On the one hand, at their positions $\boldsymbol{r} = \boldsymbol{r}_\alpha(t)$ all pedestrians α leave footprints on the ground (for example, by trampling down some vegetation). Their intensity is assumed to be

$$I(\boldsymbol{r})\left[1 - \frac{\mathrm{G}(\boldsymbol{r}, t)}{\mathrm{G}_{\max}(\boldsymbol{r})}\right] , \tag{14}$$

as the clarity of a trail is limited to a maximum value $\mathrm{G}_{\max}(\boldsymbol{r})$. This causes the saturation effect

$$1 - \frac{\mathrm{G}(\boldsymbol{r}, t)}{\mathrm{G}_{\max}(\boldsymbol{r})} \tag{15}$$

of modification of the ground by new footprints. On the other hand, the ground structure changes because of regeneration of the vegetation. This will lead to a restoration of the natural ground conditions $\mathrm{G}_0(\boldsymbol{r})$ with a certain weathering rate $1/T(\boldsymbol{r})$ which is related to the durability $T(\boldsymbol{r})$ of trails. Thus the equation of environmental changes reads

$$\frac{\mathrm{dG}(\boldsymbol{r}, t)}{\mathrm{d}t} = \frac{1}{T(\boldsymbol{r})}[\mathrm{G}_0(\boldsymbol{r}) - \mathrm{G}(\boldsymbol{r}, t)] + I(\boldsymbol{r})\left[1 - \frac{\mathrm{G}(\boldsymbol{r}, t)}{\mathrm{G}_{\max}(\boldsymbol{r})}\right] \sum_\alpha \delta[\boldsymbol{r} - \boldsymbol{r}_\alpha(t)] , \tag{16}$$

where $\delta(\boldsymbol{r} - \boldsymbol{r}_\alpha)$ denotes the Dirac delta function (which yields a contribution only for $\boldsymbol{r} = \boldsymbol{r}_\alpha$).

We will now specify how the ground structure influences pedestrian motion. A trail segment at place $\boldsymbol{r}$ motivates a pedestrian at place $\boldsymbol{r}_\alpha$ to move in the direction $(\boldsymbol{r} - \boldsymbol{r}_\alpha)/\|\boldsymbol{r} - \boldsymbol{r}_\alpha\|$. However, its attractiveness decreases with the distance $\|\boldsymbol{r} - \boldsymbol{r}_\alpha(t)\|$. This has been taken into account by a factor

$$\frac{\exp[-\|\boldsymbol{r} - \boldsymbol{r}_\alpha\|/\sigma(\boldsymbol{r}_\alpha)]}{\int \mathrm{d}^2 r \exp[-\|\boldsymbol{r} - \boldsymbol{r}_\alpha\|/\sigma(\boldsymbol{r}_\alpha)]} = \frac{\exp[-\|\boldsymbol{r} - \boldsymbol{r}_\alpha\|/\sigma(\boldsymbol{r}_\alpha)]}{2\pi[\sigma(\boldsymbol{r}_\alpha)]^2} , \tag{17}$$

where $\sigma(\boldsymbol{r}_\alpha)$ characterizes visibility (so that $1/\sigma$ corresponds to the roughness of the ground). The overall attractive effect of available trails is obtained by integration of the resulting function over the green area:

$$\boldsymbol{f}_{\mathrm{tr}}(\boldsymbol{r}_\alpha, t) = \int \mathrm{d}^2 r \frac{\exp[-\|\boldsymbol{r} - \boldsymbol{r}_\alpha\|/\sigma(\boldsymbol{r}_\alpha)]}{2\pi[\sigma(\boldsymbol{r}_\alpha)]^2} \mathrm{G}(\boldsymbol{r}, t) \frac{\boldsymbol{r} - \boldsymbol{r}_\alpha}{\|\boldsymbol{r} - \boldsymbol{r}_\alpha\|} . \tag{18}$$

On homogeneous ground, the walking direction $\boldsymbol{e}_\alpha(\boldsymbol{r}_\alpha, t)$ of a pedestrian α at place $\boldsymbol{r}_\alpha(t)$ is given by the direction of the next destination $\boldsymbol{d}_\alpha$, that is,

$$\boldsymbol{e}_\alpha^0(\boldsymbol{r}_\alpha) = \frac{\boldsymbol{d}_\alpha - \boldsymbol{r}_\alpha}{\|\boldsymbol{d}_\alpha - \boldsymbol{r}_\alpha\|} . \tag{19}$$

However, because the choice of the walking direction $\boldsymbol{e}_\alpha$ is influenced by the destination and the existing trails simultaneously, we assumed the orientation relation

$$\boldsymbol{e}_\alpha(\boldsymbol{r}_\alpha, t) = \frac{\boldsymbol{e}_\alpha^0(\boldsymbol{r}_\alpha) + \boldsymbol{f}_{\mathrm{tr}}(\boldsymbol{r}_\alpha, t)}{\|\boldsymbol{e}_\alpha^0(\boldsymbol{r}_\alpha) + \boldsymbol{f}_{\mathrm{tr}}(\boldsymbol{r}_\alpha, t)\|} . \tag{20}$$

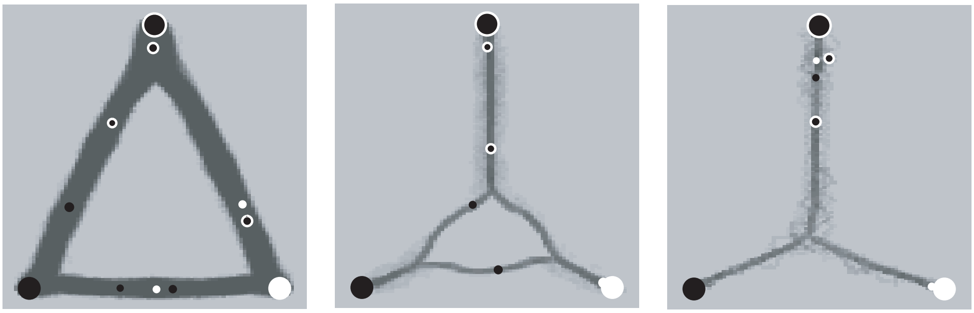
This reflects the pedestrians' compromising behavior, as it specifies the walking direction [which appears in equation (3)] by the average of the direction e_α^0 to the destination and the most attractive walking direction $\boldsymbol{e}_{\mathrm{tr}} = \boldsymbol{f}_{\mathrm{tr}}/\|\boldsymbol{f}_{\mathrm{tr}}\|$.

In cases of rare interactions, the motion of a pedestrian α with desired velocity $v_\alpha^0 \approx v^0$ is simply given by

$$\frac{\mathrm{d}\boldsymbol{r}_\alpha}{\mathrm{d}t} = v^0 \boldsymbol{e}_\alpha(\boldsymbol{r}_\alpha, t) + \boldsymbol{\zeta}_\alpha(t) , \tag{21}$$

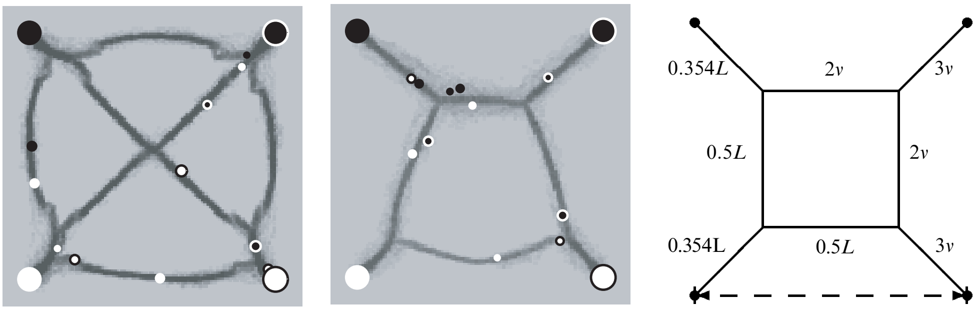
where $\boldsymbol{\zeta}_\alpha(t)$ reflects individual velocity variations.

By appropriate scaling, the above equations can be transformed into dimensionless equations which include only two independent parameters: $\kappa = IT/\sigma^2$ and $\lambda = v^0 T/\sigma$. Nevertheless the active walker model of human trail formation compares well with



(a) (b) (c)

Figure 16. The structure of the emerging trail system (dark gray) essentially depends on the persistence parameter κ. If κ is large, a direct way system results (a). If κ is small, a minimal way system is formed (c). Otherwise a compromise between both extremes will develop (b), which looks similar to the main trail system in the center of figure 14(a). In the above three pictures all pedestrians (small disks) are represented by the same symbols as their respective destinations (large disks).



(a) (b) (c)

Figure 17. The left and middle graphics illustrate the formation of trail systems according to our simulations. Small disks represent pedestrians on their way, and large disks their different entry points and destinations. The pedestrians use, with a frequency v, each of the six connections between the four entry points and destinations in the corners of a square with edge length L. Starting with a spatially homogeneous ground, the chosen ways change considerably over time. In the beginning, pedestrians take the direct ways. As frequently used trails become more comfortable, a bundling of trails sets in (a) which eventually reduces the overall length of the trail system (b). The resulting way system could serve as a planning guideline (c). It provides a suitable compromise between small construction costs and a large comfort of walking. At an overall length which is 50% shorter than the direct way system, it requires everyone to take a relative detour of 21%, which is a fair and acceptable solution. Moreover, the resulting trail system is structurally stable under the assumption that a frequency $1.5v$ of usage is needed to support permanent trails in competition with the weathering effect. If the frequency of usage is higher than this threshold, the outcome is a direct way system similar to the picture on the left.

empirical findings (Helbing, 1997; 1998; Helbing et al, 1997a; 1997b). This is illustrated, for example, by comparing the triangular island in figure 16(b) with the one in figure 14(a).

Our simulations begin with a spatially homogeneous ground. At a certain rate, pedestrians move between given entry points and destinations, starting at a random point in time. In figure 16 (figure 17) pedestrians move between all possible pairs of three (four) fixed places. At the beginning, pedestrians take the direct routes to their respective destinations because there is no reason to choose another route. However, after some time they begin to use existing paths because this is more comfortable than clearing new ways. In this way, a kind of selection process between trails sets in (compare with Schweitzer and Schimansky-Geier, 1994). On the one hand, frequently used trails are more comfortable and therefore more attractive than others. For this reason they are chosen very often. The resulting reinforcement makes them even more attractive, until the saturation effect becomes effective (owing to the limitation of walking comfort given by $G_{\max}$). On the other hand, rarely used trails are destroyed by the weathering effect. This limits the maximum length of the way system which can be supported by a certain rate of trail usage. As a consequence, the trails begin to bundle, especially where different paths meet or intersect. Finally, pedestrians with different destinations use and support common parts of the trail system, which explains the empirically found deviations from direct way systems [figures 16(b) and 17(b)].

A direct way system (which provides the shortest connections but covers a lot of space) develops only in cases of high frequencies of usage [figures 16(a) and 17(a)]. If the persistence κ of existing trails is small (as for rapidly regenerating ground), the final trail system is a minimal way system (which is the shortest way system that connects all entry points and destinations) [figure 16(c)]. For realistic values of κ, the evolution of the trail system stops before this state is reached [figure 16(b)]. Thus κ is related to the average relative detour of the walkers.

4.2 Optimization of way systems

The trail systems resulting from the mechanisms described above are particularly suitable as planning guidelines for the construction of optimal way systems. First, they take into account the walking and orientation habits, so that pedestrians will actually use such ways. Second, the resulting trail systems provide the best compromise between maximum shortness and route comfort. Third, they seem to offer fair solutions, which balance the relative detours of all pedestrians [figure 17(c)].

Computer simulations of this kind can be used for answering various questions, given a knowledge of the entry points and destinations as well as the expected rates of usage of the corresponding connections, which can be determined by established models (Borgers and Timmermans, 1986a; 1986b).

(1) Which is the trail system that pedestrians would naturally use?
Solution: Simulate the problem with realistically chosen values of λ and κ.
(2) Is the resulting way system structurally stable with respect to small changes in parameters λ and κ (for example, because of varying weather conditions)?
Solution: Simulate the system with slightly modified parameter values and check whether the topological structure of the trail system changes.
(3) Given a certain amount of money to build a way system of a certain length, which way system should be built, that is, which one is most comfortable or 'intelligent'?
Solution: Control the overall length of the evolving way system by variation of κ, until it fits the desired length.

(4) If a certain level of comfort is to be provided, which is the cheapest way system satisfying this demand?
Solution: Increase κ, starting with small values, until pedestrians take the average relative detour, which was specified to be acceptable.
(5) Given an existing way system, how should it be extended?
Solution: Take into account the existing way system by setting $G_0(\boldsymbol{r}) = G_{max}$ and check where the resulting way system contains additional trails.

Owing to the large number of walkers and the different timescales involved, the multiagent simulations of trail formation proposed above are rather time-consuming (taking about an hour of simulation time). Therefore it is reasonable to derive macroscopic equations of trail formation from the microscopic ones discussed here. As the final trail systems correspond to the stationary solutions of these macroscopic equations, they can be obtained by a simple and fast iteration scheme. A detailed discussion of this issue is presented by Helbing et al (1997b).

5 Conclusions

It was pointed out that pedestrian dynamics shows various collective phenomena, for example, lane formation and oscillatory flows through bottlenecks. These and other empirical findings can be described realistically by microscopic simulations of pedestrian streams which are based on a behavioral force model. According to this model, the collective patterns of motion can be interpreted as self-organization phenomena, arising from the nonlinear interactions among pedestrians.

We point out that self-organization flow patterns can significantly change the capacities of pedestrian facilities. They often lead to undesirable obstructions, but they can also be utilized to obtain more efficient pedestrian flows with less space. Applications in the optimization of pedestrian facilities are therefore quite natural.

In addition, improvements of way systems can be worked out with an active walker model of human trail formation. This includes additional indirect interactions between pedestrians which are caused by environmental changes and their influence on human walking behavior.

Acknowledgements. Most of this work has been carried out during our time at the II. Institute of Theoretical Physics, Pfaffenwaldring 57/III, 70550 Stuttgart, Germany. Dirk Helbing would like to thank DFG for financial support (Heisenberg scholarship He 2789/1-1). Moreover, the authors are grateful to Joachim Keltsch and Tamás Vicsek for fruitful collaboration. Finally they thank Gordon and Breach, Shaker, Springer, and World Scientific publishers for their kind permission to reprint parts of published material (see Helbing, 1996; 1997; 1998; Molnár, 1996; Helbing and Molnár, 1997).

References

Arns T, 1993, Video films of pedestrian crowds, Wannenstrasse 22, 70199 Stuttgart

Ashford N, O'Leary M, McGinity P D, 1976, "Stochastic modelling of passenger and baggage flows through an airport terminal" *Traffic Engineering and Control* **17** 207 – 210

Baeck T, 1996 *Evolutionary Algorithms in Theory and Practice* (Oxford University Press, New York)

Batty M, 1997, "Predicting where we walk" *Nature* **388** 19 – 20

Boeminghaus D, 1982 *Fußgängerbereiche + Gestaltungselemente/Pedestrian Areas and Design Elements/Zones pour Piétons + Élements de Conception* (Krämer, Stuttgart)

Bolay K, 1998, "Nichtlineare Phänomene in einem fluid-dynamischen Verkehrsmodell" [Nonlinear phenomena in a fluid-dynamic traffic model], Master's thesis, Department of Theoretical Physics II, University of Stuttgart, Stuttgart

Borgers A, Timmermans H, 1986a, "A model of pedestrian route choice and demand for retail facilities within inner-city shopping areas" *Geographical Analysis* **18** 115 – 128

Borgers A, Timmermans H, 1986b, "City centre entry points, store location patterns and pedestrian route choice behaviour: a microlevel simulation model" *Socio-Economic Planning Science* **20** 25 – 31

Brilon W, Großmann M, Blanke H, 1993, "Verfahren für die Berechnung der Leistungsfähigkeit und Qualität des Verkehrsablaufes auf Straßen" [Methods for the calculation of the capacity and quality of traffic flow in streets], in Straßenbau und Straßenverkehrstechnik series number 669, Ministry of Traffic, Bonn, chapter 13
Carstens R L, Ring S L, 1970, "Pedestrian capacities of shelter entrances" *Traffic Engineering* **41** 38 – 43
Coleman J S, 1964 *Introduction to Mathematical Sociology* (Free Press, New York) pp 361 – 375
Coleman J S, James J, 1961, "The equilibrium size distribution of freely-forming groups" *Sociometry* **24** 36 – 45
Davis D G, Braaksma J P, 1988, "Adjusting for luggage-laden pedestrians in airport terminals" *Transportation Research A* **22** 375 – 388
Drager K H, Løvås G, Wiklund J, Soma H, Duong D, Violas A, Lanèrès V, 1992, "EVACSIM: a comprehensive evacuation simulation tool", in *Proceedings of the 1992 Emergency Management and Engineering Conference* (Society for Computer Simulation, Orlando, FL) pp 101 – 108
Duparcmeur Y L, Herrmann H, Troadec J P, 1995, "Spontaneous formation of vortex in a system of self motorised particles" *Journal de Physique I France* **5** 1119 – 1128
Ebihara M, Ohtsuki A, Iwaki H, 1992, "A model for simulating human behavior during emergency evacuation based on classifactory reasoning and certainty value handling" *Microcomputers in Civil Engineering* **7** 63 – 71
Fruin J J, 1971, "Designing for pedestrians: a level-of-service concept", in *Highway Research Record* Number 355: Pedestrians (Highway Research Board, Washington, DC) pp 1 – 15
Ganem J, 1998, "A behavioral demonstration of Fermat's principle" *The Physics Teacher* **36** 76 – 78
Garbrecht D, 1973, "Describing pedestrian and car trips by transition matrices" *Traffic Quarterly* **27** 89 – 109
Gipps P G, Marksjö B, 1985, "A micro-simulation model for pedestrian flows" *Mathematics and Computers in Simulation* **27** 95 – 105
Gopal S, Smith T R, 1990, "NAVIGATOR: an AI-based model of human way-finding in an urban environment", in *Spatial Choices and Processes* Eds M M Fischer, P Nijkamp, Y Y Papageorgiou (North-Holland, Amsterdam) pp 169 – 200
Hankin B D, Wright R A, 1958, "Passenger flow in subways" *Operational Research Quarterly* **9** 81 – 88
Helbing D, 1991, "A mathematical model for the behavior of pedestrians" *Behavioral Science* **36** 298 – 310
Helbing D, 1992a, "A fluid-dynamic model for the movement of pedestrians" *Complex Systems* **6** 391 – 415
Helbing D, 1992b, "Models for pedestrian behavior", in *Natural Structures* Part II (Sonderforschungsbereich 230, Stuttgart) pp 93 – 98
Helbing D, 1993 *Stochastische Methoden, nichtlineare Dynamik und quantitative Modelle sozialer Prozesse* [Stochastic methods, non-linear dynamics and quantitative models of social processes] (Shaker, Aachen)
Helbing D, 1994, "A mathematical model for the behavior of individuals in a social field" *Journal of Mathematical Sociology* **19** 189 – 219
Helbing D, 1995 *Quantitative Sociodynamics: Stochastic Methods and Models of Social Interaction Processes* (Kluwer, Dordrecht)
Helbing D, 1996, "Traffic modeling by means of physical concepts", in *Traffic and Granular Flow* Eds D E Wolf, M Schreckenberg, A Bachem (World Scientific, Singapore) pp 87 – 104
Helbing D, 1997 *Verkehrsdynamik* [Traffic dynamics] (Springer, Berlin)
Helbing D, 1998, "Pedestrian dynamics and trail formation", in *Traffic and Granular Flow '97* Eds M Schreckenberg, D E Wolf (Springer, Berlin) pp 21 – 36
Helbing D, Molnár P, 1995, "Social force model of pedestrian dynamics" *Physical Review E* **51** 4282 – 4286
Helbing D, Molnár P, 1997, "Self-organization phenomena in pedestrian crowds", in *Self-organization of Complex Structures: From Individual to Collective Dynamics* Ed. F Schweitzer (Gordon and Breach, London) pp 569 – 577
Helbing D, Tilch B, 1998, "Generalized force model of traffic dynamics" *Physical Review E* **58** 133 – 138
Helbing D, Vicsek T, 1999, "Optimal self-organization" *New Journal of Physics* **1** 13.1 – 13.17
Helbing D, Molnár P, Schweitzer F, 1994, "Computer simulations of pedestrian dynamics and trail formation", in *Evolution of Natural Structures* (Sonderforschungsbereich 230, Stuttgart) pp 229 – 234

Helbing D, Keltsch J, Molnár P, 1997a, “Modelling the evolution of human trail systems” *Nature* **388** 47 – 50
Helbing D, Schweitzer F, Keltsch J, Molnár P, 1997b, “Active walker model for the formation of human and animal trail systems” *Physical Review E* **56** 2527 – 2539
Helbing D, Farkas I, Vicsek T, 2000, “Freezing by heating in a driven mesoscopic system” *Physical Review Letters* **84** 1240 – 1243
Henderson L F, 1971, “The statistics of crowd fluids” *Nature* **229** 381 – 383
Henderson L F, 1974, “On the fluid mechanics of human crowd motion” *Transportation Research* **8** 509 – 515
Henderson L F, Jenkins D M, 1973, “Response of pedestrians to traffic challenge” *Transportation Research* **8** 71 – 74
Henderson L F, Lyons D J, 1972, “Sexual differences in human crowd motion” *Nature* **240** 353 – 355
Hill M R, 1984 *Walking, Crossing Streets, and Choosing Pedestrian Routes* (University of Nebraska Press, Lincoln, NE)
Hoel L A, 1968, “Pedestrian travel rates in central business districts” *Traffic Engineering* **38** 10 – 13
Humpert K, Becker S, Brenner K, Gangler A, 1996, “Planung ‘gewachsener’ Städte” [Design of ‘grown’ cities], in *Prozeß und Form ‘Natürlicher Konstruktionen’* [Process and form of ‘natural constructions’] Eds K Teichmann, J Wilke (Ernst and Sohn, Berlin) pp 172 – 181
Kadanoff L P, 1985, “Simulating hydrodynamics: a pedestrian model” *Journal of Statistical Physics* **39** 267 – 283
Keltsch J, 1996, “Selbstorganisation von Wegen durch ‘Active Walkers’ ” [Self-organization of way systems by ‘active walkers’] Master’s thesis, Department of Theoretical Physics II, University of Stuttgart, Stuttgart
Kirsch H, 1964, “Leistungsfähigkeit und Dimensionierung von Fußgängerwegen” [Capacity and dimensioning of pedestrian footways], Straßenbau und Straßenverkehrstechnik series number 33, Ministry of Traffic, Bonn
Klockgether J, Schwefel H-P, 1970, “Two-phase nozzle and hollow core jet experiments”, in *Proceedings of the Eleventh Symposium Engineering Aspects of Magnetohydrodynamics* Ed. D G Elliott, California Institute of Technology, Pasadena, CA, pp 141 – 148
Lewin K, 1951 *Field Theory in Social Science* (Harper and Brothers, New York)
Løvås G G, 1994, “Modelling and simulation of pedestrian traffic flow” *Transportation Research B* **28** 429 – 443
Makse H A, Havlin S, King P R, Stanley H E, 1997, “Spontaneous stratification in granular mixtures” *Nature* **386** 379 – 382
Mayne A J, 1954, “Some further results in the theory of pedestrians and road traffic” *Biometrika* **41** 375 – 389
Molnár P, 1996 *Modellierung und Simulation der Dynamik von Fußgängerströmen* [Modelling and simulation of the dynamics of pedestrian motion] (Shaker, Aachen)
Mōri M, Tsukaguchi H, 1987, “A new method for evaluation of level of service in pedestrian facilities” *Transportation Research A* **21** 223 – 234
Muramatsu M, Irie T, Nagatani T, 1999, “Jamming transition in pedestrian counter flow” *Physica A* **267** 487 – 498
Navin P D, Wheeler R J, 1969, “Pedestrian flow characteristics” *Traffic Engineering* **39** 31 – 36
Oeding D, 1963, “Verkehrsbelastung und Dimensionierung von Gehwegen und anderen Anlagen des Fußgängerverkehrs” [Traffic volume and dimensioning of footways and other facilities of pedestrian traffic], Straßenbau und Straßenverkehrstechnik series number 22, Ministry of Traffic, Bonn
O’Flaherty C A, Parkinson M H, 1972, “Movement on a city centre footway” *Traffic Engineering and Control* **13** 434 – 438
Older S J, 1968, “Movement of pedestrians on footways in shopping streets” *Traffic Engineering and Control* **10** 160 – 163
Pauls J, 1984, “The movement of people in buildings and design solutions for means of egress” *Fire Technology* **20** 27 – 47
Pennec T L, Måløy K J, Hansen A, Ammi M, Bideau D, Wu X-L, 1996, “Ticking hour glasses: experimental analysis of intermittent flow” *Physical Review E* **53** 2257 – 2264
Polus A, Schofer J L, Ushpiz A, 1983, “Pedestrian flow and level of service” *Journal of Transportation Engineering* **109** 46 – 56
Predtetschenski W M, Milinski A I, 1971 *Personenströme in Gebäuden: Berechnungsmethoden für die Projektierung* [Pedestrian flow in buildings: calculation methods for design] (Müller, Köln-Braunsfeld)

Rechenberg I, 1973, "Evolutionsstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution" [Evolutionary strategy: optimization of technical systems according to principles of biological evolution] (Frommann-Holzboog, Stuttgart)
Reynolds C W, 1987, "Flocks, herds, and schools: a distributed behavioral model" *Computer Graphics* **21** 25 – 34
Reynolds C W, 1994, "Evolution of corridor following behavior in a noisy world", in *From Animals to Animats 3: Proceedings of the Third International Conference on Simulation of Adaptive Behavior (SAB94)* Eds D Cliff, P Husbands, J-A Meyer, S Wilson (MIT Press, Cambridge, MA) pp 402 – 410
Reynolds C W, 1999, "Steering behaviors for autonomous characters", in *Proceedings of the 1999 Game Developer's Conference* Ed. A Yu (Miller Freeman, San Francisco, CA) pp 763 – 782
Roy J R, 1992, "Queuing in spatially dispersed public facilities", paper presented at the Fourth World Congress of the Regional Science Association International, Palma de Mallorca; copy available from the authors
Sandahl J, Percivall M, 1972, "A pedestrian traffic model for town centers" *Traffic Quarterly* **26** 359 – 372
Santra S B, Schwarzer S, Herrmann H, 1996, "Fluid-induced particle segregation in sheared granular assemblies" *Physical Review E* **54** 5066 – 5072
Schaur E, 1991 *Ungeplante Siedlungen* [Non-planned Settlements] (Krämer, Stuttgart)
Schelhorn T, O'Sullivan D, Haklay M, Thurstain-Goodwin M, 1999, "STREETS: an agent-based pedestrian model", paper presented at the conference Computers in Urban Planning and Urban Management, Venice; copy available from the authors
Schubert H, 1967, "Planungsmaßnahmen für den Fußgängerverkehr in den Städten" [Planning measures for pedestrian traffic in cities], Straßenbau und Straßenverkehrstechnik series number 56, Ministry of Traffic, Bonn
Schweitzer F, Schimansky-Geier L, 1994, "Clustering of 'active' walkers in a two-component system" *Physica A* **206** 359 – 379
Still G K, 1993, "New computer system can predict human behavior response to building fires" *Fire* **84** 40 – 41
Timmermans H, van der Hagen X, Borgers A, 1992, "Transportation systems, retail environments and pedestrian trip chaining behaviour: modelling issues and applications" *Transportation Research B* **26** 45 – 59
TRB, 1985, "Pedestrians", in *Highway Capacity Manual* special report 209 (Transportation Research Board, Washington, DC) chapter 13
Vicsek T, Czirók A, Ben-Jacob E, Cohen I, Shochet O, 1995, "Novel type of phase transition in a system of self-driven particles" *Physical Review Letters* **75** 1226 – 1229
Virkler M R, Elayadath S, 1994, "Pedestrian density characteristics and shockwaves", in *Proceedings of the Second International Symposium on Highway Capacity* Ed. R Akçelik (Transportation Research Board, Washington, DC) volume 2, pp 671 – 684
Weidmann U, 1993, "Transporttechnik der Fußgänger" [Transportation technique for pedestrians], Schriftenreihe des Instituts für Verkehrsplanung, Transporttechnik, Straßen- und Eisenbahnbau number 90 (ETH Zürich, Switzerland)
Whyte W H, 1988 *City. Rediscovering the Center* (Doubleday, New York)
Wu X-L, Måløy K J, Hansen A, Ammi M, Bideau D, 1993, "Why hour glasses tick" *Physical Review Letters* **71** 1363 – 1366
Yuhaski S J Jr, Macgregor Smith J M, 1989, "Modelling circulation systems in buildings using state dependent queueing models" *Queueing Systems* **4** 319 – 338